

# Reconstruction of Quark Mass Matrices with Weak Basis Texture Zeroes from Experimental Input

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## Abstract

All quark mass matrices with texture zeroes obtained through weak basis transformations are confronted with the experimental data. The reconstruction of the quark mass matrices  $M_u$  and  $M_d$  at the electroweak scale is performed in a weak basis where the matrices are Hermitian and have a maximum of three vanishing elements. The same procedure is also accomplished for the Yukawa coupling matrices at the grand unification scale in the context of the Standard Model and its minimal supersymmetric extension as well as of the two Higgs doublet model. The analysis of all viable power structures on the quark Yukawa coupling matrices that could naturally appear from a Froggatt-Nielsen mechanism is also presented.

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## I. INTRODUCTION

In spite of the enormous experimental progress, the origin of the fermion mass pattern is still a fundamental question not yet solved in Particle Physics. Many theoretical attempts have been made in order to go beyond the Standard Model (SM) where a flavor symmetry would predict correctly the observed mass and mixing hierarchies. Among many proposals in the literature, flavor gauge symmetries in the context of grand unified theories (GUT), supersymmetric or not, are favourite candidates [1] toward a more fundamental theory. Examples of such possibilities are Yukawa mass matrices involving texture zeroes and/or based on a spontaneously broken Abelian symmetry [2].

Generically, in the context of the SM the quark mass matrices  $M_u$  and  $M_d$  are complex (36 real parameters) and they have to reproduce 10 physical parameters, namely the six observed quark masses and their mixing angles encoded in the Cabibbo-Kobayashi-Maskawa (CKM) unitary matrix,  $V$ . There is still some freedom left that can account for this redundancy, which corresponds to the transformation that leaves the charged currents invariant, the so-called weak basis (WB) transformation. In theories beyond the SM transformations of this type are expected to be less general. It turns out that some weak bases seem more natural than others when searching for an underlying flavor symmetry. Once the more fundamental theory is broken down to the SM, WB transformations on the light fields scramble the main properties of the flavor symmetry.

The simplest attempt at understanding the flavor structure encoded in the fermion mass matrices is by imposing some texture zeroes on the matrix elements. The existence of such zeroes strongly evokes a new symmetry which, when exact, enforces the mass matrix elements to vanish [3]. In the context of the Froggatt-Nielsen mechanism [2] or other approaches [4], a texture zero could be understood as a suppressed matrix element. The relevant question now is whether such texture zeroes do have physical implications [5] or can simply be removed through an appropriate WB transformation. Thus, given the matrices  $M_u$  and  $M_d$ , it is essential to distinguish zeroes which have no physical content in themselves among others that imply physical constraints on the parameter space.

Taking into account the awesome improvement in the experimental determination of the CKM matrix [6], many texture zero structures found in the literature [3, 4] are not consistent with the observed data. The challenges of a model that makes an attempt to

reproduce the data arise from the precise determination of the rephasing invariant angle  $\beta \equiv \arg(-V_{cd}V_{cb}^*V_{td}^*V_{tb})$ , which is rather constrained [7], and  $\gamma \equiv \arg(-V_{ud}V_{ub}^*V_{cd}^*V_{cb})$ . In spite of the large experimental errors, the measurement of  $\gamma$  [8, 9] is determinant due to the fact that its extraction from input data is essentially not affected by the presence of new physics contributions to  $B_d^0 - \bar{B}_d^0$  and  $B_s^0 - \bar{B}_s^0$  mixings [10].

The aim of this paper is the reconstruction of the quark mass matrices at the electroweak and GUT scales in weak bases where they are Hermitian and have a maximal number of non-physical zeroes. We also address the question whether the reconstructed matrices at GUT scale can contain more zeroes with physical content and eventually could exhibit a Froggatt-Nielsen flavor symmetry. Another ambitious task of our bottom-up approach is the systematic search for allowed Hermitian quark Yukawa structures at GUT scale.

This paper is organized as follows. In Section II, we show that starting from arbitrary matrices  $M_u$  and  $M_d$ , it is always possible to perform a WB transformation that renders them Hermitian, with a common zero located at the  $(1, 1)$  element and a zero in the position  $(1, 3)$  in the down quark sector. We then prove that only three WB zeroes in the mass matrices are allowed. In Section III, we confront the obtained quark matrices with the present experimental data, reconstructing them at the electroweak scale and at a high scale where the Froggatt-Nielsen mechanism can be implemented. Finally, in the last section we draw our conclusions.

## II. WEAK BASIS TEXTURE ZEROES

In this section, we review the proof that it is always possible to make a WB transformation such that the quark mass matrices are Hermitian and have the following form [5]

$$M_u = \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \quad (1)$$

The proof is rather straightforward if one starts from the basis where the up mass matrix,  $M_u^0$ , is diagonal and the down mass matrix,  $M_d^0$ , is Hermitian,

$$\begin{aligned} M_u^0 &= \text{diag}(m_u, m_c, m_t), \\ M_d^0 &= V \text{diag}(m_d, m_s, m_b) V^\dagger. \end{aligned} \quad (2)$$

Then we look for a weak basis transformation which transforms the pair of matrices written in Eq. (2) into the pair in Eq. (1),

$$\begin{aligned} M_u^0 \rightarrow M_u &= \mathcal{W}^\dagger M_u^0 \mathcal{W}, \\ M_d^0 \rightarrow M_d &= \mathcal{W}^\dagger M_d^0 \mathcal{W}, \end{aligned} \quad (3)$$

where  $\mathcal{W}$  is a unitary matrix which, according to Eq. (3) and Eq. (2), has to satisfy the following condition:

$$(M_u)_{11} = (M_d)_{11} = (M_d)_{13} = 0. \quad (4)$$

In order to make clear the existence of the unitary matrix  $\mathcal{W}$ , we have split the action of  $\mathcal{W}$  in two factors,

$$\mathcal{W} = W W', \quad (5)$$

and required the unitary matrix  $W$  to acquire the zeroes at the position  $(1, 1)$  in both quark sectors. The unitary matrix  $W'$  is then used to get the third zero at the position  $(1, 3)$  in the down quark sector, without destroying the zeroes made through the WB transformation  $W$ .

Demanding zeroes at the position  $(1, 1)$  implies that the WB transformation  $W$  obeys the following set of equations [5]:

$$m_u |W_{11}|^2 + m_c |W_{21}|^2 + m_t |W_{31}|^2 = 0, \quad (6a)$$

$$m_d |X_{11}|^2 + m_s |X_{21}|^2 + m_b |X_{31}|^2 = 0, \quad (6b)$$

$$|W_{11}|^2 + |W_{21}|^2 + |W_{31}|^2 = 1, \quad (6c)$$

where  $X$  is given by  $X \equiv V^\dagger W$  and  $|X_{i1}|^2$  for  $i = 1, 2, 3$  is given by

$$\begin{aligned} |X_{i1}|^2 &= |V_{1i}|^2 |W_{11}|^2 + |V_{2i}|^2 |W_{21}|^2 + |V_{3i}|^2 |W_{31}|^2 \\ &\quad + 2\text{Re}(V_{1i}^* W_{11} V_{2i} W_{21}) + 2\text{Re}(V_{1i}^* W_{11} V_{3i} W_{31}^*) \\ &\quad + 2\text{Re}(V_{2i}^* W_{21} V_{3i} W_{31}^*). \end{aligned}$$

Note however that, in order to have a solution for the system of Eqs. (6a)-(6c), one of the quark masses in each sector must have an opposite sign to the other two. This requirement can always be fulfilled, since the sign of a Dirac fermion mass can be fixed through an appropriate chiral transformation. Thus, without loss of generality one can always restrict to the case where only one mass is negative and the other two positive. For convenience we

write the WB transformation  $W$  as

$$\begin{aligned} W_{11} &= \cos \rho \cos \eta e^{i\varphi_1}, \\ W_{21} &= \cos \rho \sin \eta e^{i\varphi_2}, \\ W_{31} &= \sin \rho e^{i\varphi_3}, \end{aligned} \quad (7)$$

where the angles  $\rho$  and  $\eta$  are chosen in the first quadrant. Note that with this choice the unitarity constraint given in Eq. (6c) is automatically verified. Once the angle  $\rho$  is given, the angle  $\eta$  is simply determined from Eq. (6a) as

$$\tan^2 \eta = -\frac{m_u \cos^2 \rho + m_c \sin^2 \rho}{m_t}. \quad (8)$$

The interval for the angle  $\rho$  varies conforming to the sign of the up quark masses and is restricted to

$$\rho \in \left[ \arctan \sqrt{\frac{m_u}{-m_c}}, \frac{\pi}{2} \right], \quad (9a)$$

when  $m_c$  is negative and

$$\rho \in \left[ 0, \arctan \sqrt{\frac{-m_u}{m_c}} \right], \quad (9b)$$

when  $m_u$  is negative. Finally, Eq. (6b) yields a relation among the phases  $\varphi_1$ ,  $\varphi_2$  and  $\varphi_3$ . We also remark that there is an infinite choice of unitary matrices  $W$  with the first column given by Eqs. (7). Such indetermination can always be seen as a redefinition of the parameters of the unitary matrix  $W'$ .

The existence of the third zero at the position  $(1, 3)$  is assured by the WB transformation  $W'$ , which can be parametrized as

$$W' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -e^{i\varphi} \sin \theta \\ 0 & e^{-i\varphi} \sin \theta & \cos \theta \end{pmatrix}, \quad (10)$$

and do not change clearly the zeroes at the entries  $(1, 1)$ . The condition for having the third zero,

$$(M_d)_{13} = 0, \quad (11)$$

implies that the angle  $\theta$  is

$$\tan \theta = \left| \frac{(W^\dagger M_d^0 W)_{13}}{(W^\dagger M_d^0 W)_{12}} \right|, \quad (12)$$

and the phase  $\varphi$  is

$$\varphi = \arg \left[ \frac{(W^\dagger M_d^0 W)_{13}}{(W^\dagger M_d^0 W)_{12}} \right]. \quad (13)$$

Now we address the question whether it is possible to keep both quark mass matrices Hermitian and, simultaneously, obtain additional zeroes through WB transformations. By counting the number of parameters present in the mass matrices of Eq. (1), one gets twelve independent real parameters, which are more than the ten physical parameters. In principle, one could use such freedom to perform a WB transformation in order to have more than three zeroes. Such a WB transformation is however not possible, thus implying that the assumption of any additional zero does have now physical implications [5]. On the other hand, if one relaxes the assumption of Hermiticity of the quark mass matrices, it can be shown that more zeroes can be obtained through a WB transformation, *e.g.* in the so-called parallel non-Hermitian nearest neighbor interactions basis [11].

Another question one may raise is the possibility of having the WB zeroes located at different positions than the WB zeroes given in Eq. (1). For example, instead of having the third zero located at (1, 3), it is possible to make a WB transformation on the mass matrices in order to obtain the third zero at the position (1, 2), (2, 2), (2, 3) or (3, 3). To get the zero at the position (2, 2) we can perform WB transformation,

$$\mathcal{W}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon & -\sin \varepsilon \\ 0 & \sin \varepsilon & \cos \varepsilon \end{pmatrix}, \quad (14)$$

with  $\varepsilon$  given by

$$\tan \varepsilon = \frac{M_{d23} \pm \sqrt{M_{d23}^2 - M_{d22} M_{d33}}}{M_{d33}}, \quad (15)$$

after rendering  $M_d$  real in Eq. (1). If instead we consider the third zero at the position (2, 3), the angle  $\varepsilon$  is then given by

$$\tan \varepsilon = \frac{M_{d33} - M_{d22} \pm \sqrt{(M_{d33} - M_{d22})^2 + 4M_{d23}^2}}{2M_{d23}}. \quad (16)$$

Thus, starting from the quark mass matrices given in Eq. (1) one can always obtain two new pairs of mass matrices with the third zero at the position (2, 2) or (2, 3).

The above three pairs of mass matrices share the property of having the zeroes at the same position (1, 1) for both quark sectors. There is still the possibility to move the positions

of the zeroes through new WB transformations of the form

$$\begin{aligned} M'_u &= P_i^T M_u P_i, \\ M'_d &= P_i^T M_d P_i, \end{aligned} \tag{17}$$

where  $P_i$ ,  $i = 1, \dots, 6$  are the six real permutation matrices, isomorphic to  $S_3$ . We conclude that one can derive through WB transformations all pairs  $M_u$  and  $M_d$  (15 pairs altogether), having one zero at the same position in the diagonal for both sectors and an additional zero in the down-quark sector. We could easily adapt the WB transformation  $W'$  in order to reproduce the extra zero in the up sector instead of the down-quark sector, extending these WB zero textures to a total of 30 pairs. This procedure can also be implemented in the leptonic sector [12].

### III. CONFRONTING THE WEAK BASIS ZEROES WITH EXPERIMENTAL DATA

The main goal of this paper is the reconstruction of the quark mass matrices  $M_u$  and  $M_d$  in a basis where they are Hermitian with zeroes at the same positions in the diagonal and an additional zero located in  $M_d$ . In this analysis we selected, among other equivalent possibilities, the mass matrices with the common zero at the  $(1, 1)$  element and the additional zero at the position  $(1, 3)$ . This reconstruction is performed first at the electroweak scale,  $M_Z \simeq 91.2$  GeV, and later at GUT scale,  $\Lambda$ .

#### A. The Quark Mass Matrices Reconstructed at $M_Z$

We reconstruct the quark mass matrices in the weak basis given in Eq. (1), starting from Eq. (2) and implementing the procedure described in Section II. In order to construct the physical basis given in Eq. (2) at  $M_Z$ , we have run all the quark masses up to  $M_Z$  in the  $\overline{MS}$

scheme,

$$m_u(M_Z) = 1.4_{-0.5}^{+0.6} \text{ MeV}, \quad (18)$$

$$m_d(M_Z) = 2.8 \pm 0.7 \text{ MeV}, \quad (19)$$

$$m_s(M_Z) = 60_{-19}^{+15} \text{ MeV}, \quad (20)$$

$$m_c(M_Z) = 0.64_{-0.09}^{+0.07} \text{ GeV}, \quad (21)$$

$$m_b(M_Z) = 2.89_{-0.08}^{+0.17} \text{ GeV}, \quad (22)$$

$$m_t(M_Z) = 170.1 \pm 2.3 \text{ GeV}, \quad (23)$$

using the renormalization group equations (RGE) for QCD [13] at three loops. The input masses employed in this RGE programme are summarized in Appendix A. For the quark mixing matrix, we have constructed the unitary CKM matrix by taking the following mixing angles [7]

$$|V_{us}| = 0.2255 \pm 0.0019, \quad (24)$$

$$|V_{ub}| = (3.93 \pm 0.36) \times 10^{-3}, \quad (25)$$

$$|V_{cb}| = (41.2 \pm 1.1) \times 10^{-3}. \quad (26)$$

In order to fully build the CKM matrix one needs a fourth parameter that we choose to be a CP violating quantity from the unitarity triangle - a useful graphical representation of the unitarity relation between the first and the third column of the CKM matrix. Among many possible choices, we have considered either the angle  $\beta$  [7],

$$\sin 2\beta = 0.681 \pm 0.025, \quad (27)$$

which is rather constrained, or the angle  $\gamma$  [7],

$$\gamma = (77_{-32}^{+30})^\circ. \quad (28)$$

The details of how the reconstruction of the CKM matrix was made for these two choices,  $\beta$  and  $\gamma$ , are given in Appendix A.

With the purpose of reconstructing the mass matrices  $M_u$  and  $M_d$  at  $M_Z$  in the WB of Eq. (1), we have scanned randomly the parameter space taking the experimental range of the input values given in Eqs. (18)-(28). We do not assume any correlation (*e.g.*, those arising from an alignment of the unitary matrices that diagonalize the mass matrices) among the

different matrix entries, so that our analysis is as general as possible. We have also ensured that the random input values taken from Eqs. (24)-(28) verify the experimental constraint on the Jarlskog invariant [7],

$$J \equiv \text{Im}(V_{us} V_{ub}^* V_{cs}^* V_{cb}) = (3.05^{+0.19}_{-0.20}) \times 10^{-5}. \quad (29)$$

In this reconstruction, we have taken into account all possible quark mass signs and we have varied the angle  $\rho$  from the parametrization defined in Eq. (7) according to the intervals given in Eqs. (9a)-(9b). We have also included the scanning of two unphysical phases which re-phases the CKM matrix on the left. Indeed, it is the variation of the angle  $\rho$  the main contribution for the broad range in the quark mass matrix elements. The experimentally allowed intervals for the elements of the quark matrices  $M_u$  and  $M_d$  read in GeV as

$$|M_u| = \begin{pmatrix} 0 & 0.0214 - 10.7 & 0.0137 - 2.58 \\ \dots & 0.00358 - 172 & 0.00362 - 86.5 \\ \dots & \dots & 8.87 \times 10^{-8} - 172 \end{pmatrix}, \quad (30a)$$

$$|M_d| = \begin{pmatrix} 0 & 0.00959 - 0.322 & 0 \\ \dots & 0.000146 - 3.05 & 0.0452 - 1.56 \\ 0 & \dots & 0.00270 - 3.05 \end{pmatrix}. \quad (30b)$$

One sees from these Hermitian mass matrices that it does not seem viable to have a zero at the position (1, 3) in up sector and at the same time to be compatible with the experimental data. Looking at Eq. (30a), we conclude that the only possibility for a new zero to appear, besides the three WB zeroes, is in the position (3, 3). However such a possibility is very unlikely, since in order to be consistent with the experimental data one needs an enormous fine-tuning of the parameters to maintain the WB zeroes and, simultaneously, obtain a new zero at the position (3, 3). It is even more difficult to have an additional physical zero in the case that the WB zero is located at (2, 2) or (2, 3) in the down-quark sector. The situation changes when we consider the WB zero at the position (1, 3) in the up sector. In such basis, an exact fourth zero is compatible with the experimental data as it can be seen from the

allowed range of the matrices  $M_u$  and  $M_d$  expressed in GeV,

$$|M_u| = \begin{pmatrix} 0 & 0.254 - 10.6 & 0 \\ \dots & 165 - 172 & 0 - 24.5 \\ 0 & \dots & 0.00100 - 3.04 \end{pmatrix}, \quad (31a)$$

$$|M_d| = \begin{pmatrix} 0 & 0.00116 - 0.312 & 0.000414 - 0.0171 \\ \dots & 2.75 - 3.06 & 0.0157 - 0.566 \\ \dots & \dots & 0.00190 - 0.137 \end{pmatrix}. \quad (31b)$$

We point out that the entry (2, 3) of the up-quark mass matrix given in Eq. (31a) can be exactly zero in agreement with the experimental data. One can see immediately that the unitary matrices that diagonalize the Hermitian matrices given in Eqs. (31a) and (31b) correspond to large rotations, so that the smallness of the CKM mixing angles is obtained by huge cancellations. Therefore, we conclude that at  $M_Z$  scale more than three zeroes are not compatible with the experimental data, unless we accept large rotations in the diagonalization process. This fact has been known in the literature [14, 15] and one possibility to render viable a four texture zero Ansatz is by adding an isosinglet vector-like quark which mixes with standard quarks [16].

Finally, we remark that we have not found significant differences in the resulting mass matrices if one changes the CKM reconstructed by taking, in addition to  $|V_{us}|$ ,  $|V_{ub}|$  and  $|V_{cb}|$ , either  $\sin 2\beta$  or  $\sin \gamma$ , despite the experimental observations for  $\sin \gamma$  have more uncertainties.

## B. The Quark Yukawa Coupling Matrices Reconstructed at GUT scale

Grand unified models seem a natural framework for implementing family symmetries and, in this sense, the hierarchical structure of the quark masses and mixing angles could be generated by a flavor symmetry. We examine in detail the task of reconstructing the quark Yukawa coupling matrices at the unification scale in weak bases where a common zero is in the diagonal for both sectors and an additional zero is in down-quark sector.

To fully reconstruct the Yukawa coupling matrices  $h_u$  and  $h_d$  at GUT scale,  $\Lambda$ , starting from the input values given at electroweak scale, we run the Yukawa couplings by using their 1-loop renormalization group equations [17]. Note however that, for the RGE programme from low energy to the GUT scale, we need in addition to take the charged lepton masses,

TABLE I: The allowed range for the Yukawa matrices  $h_d$  and  $h_u$  in the SM, reconstructed for three different positions of the extra WB zero in  $h_d$ , namely in the positions (1,3), (2,2) and (2,3) at  $\Lambda = 10^{14}$  GeV.

	$h_{d13} = 0$	$h_{d22} = 0$	$h_{d23} = 0$
$ h_u $	$\begin{pmatrix} 0 & \lambda^{2.4-5.7} & \lambda^{3.1-6.4} \\ \lambda^{2.4-5.7} & \lambda^{0.5-3.6} & \lambda^{0.9-6.5} \\ \lambda^{3.1-6.4} & \lambda^{0.9-6.5} & \lambda^{0.5-13.4} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.6-8.8} & \lambda^{2.4-5.0} \\ \lambda^{3.6-8.8} & \lambda^{2.5-4.8} & \lambda^{1.5-2.6} \\ \lambda^{2.4-5.0} & \lambda^{1.5-2.6} & \lambda^{0.5} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{4.2-8.9} & \lambda^{2.4-5.0} \\ \lambda^{4.2-8.9} & \lambda^{3.9-7.2} & \lambda^{2.5-3.1} \\ \lambda^{2.4-5.0} & \lambda^{2.5-3.1} & \lambda^{0.5} \end{pmatrix}$
$ h_d $	$\begin{pmatrix} 0 & \lambda^{4.8-7.0} & 0 \\ \lambda^{4.8-7.0} & \lambda^{3.3-7.2} & \lambda^{3.7-5.4} \\ 0 & \lambda^{3.7-5.4} & \lambda^{3.3-7.7} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{5.8-8.0} & \lambda^{4.8-8.4} \\ \lambda^{5.8-8.0} & 0 & \lambda^{4.5-5.1} \\ \lambda^{4.8-8.4} & \lambda^{4.5-5.1} & \lambda^{3.3} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{6.2-7.7} & \lambda^{4.8-8.2} \\ \lambda^{6.2-7.7} & \lambda^{5.6-6.9} & 0 \\ \lambda^{4.8-8.2} & 0 & \lambda^{3.3} \end{pmatrix}$

we have used their values from Particle Data Group [7]. This RGE evolution was made by considering three different effective low energy models: SM, SM extended with an extra Higgs doublet (DHM) and minimal supersymmetric extension of the SM (MSSM). In the case of the SM, the quark Yukawa coupling matrices  $h_u$  and  $h_d$  at  $M_Z$  are related to the quark mass matrices  $M_u$  and  $M_d$  through the following relations

$$h_u(M_Z) = \frac{M_u(M_Z)}{v}, \quad h_d(M_Z) = \frac{M_d(M_Z)}{v}, \quad (32)$$

where  $v = 174.1$  GeV is the vacuum expectation value (VEV) of the Higgs doublet. These relations change to

$$h_u(M_Z) = \frac{M_u(M_Z)}{v \sin \beta}, \quad h_d(M_Z) = \frac{M_d(M_Z)}{v \cos \beta}, \quad (33)$$

in the case of extending the SM with an extra Higgs doublet with opposite hypercharge. In Eq. (33) we have parametrized the VEVs with  $\beta$ , defined by the ratio  $\tan \beta \equiv v_u/v_d$ , and  $v$ , verifying  $v_u^2 + v_d^2 = v^2$ . The quantities  $v_u$  and  $v_d$  are the VEVs of Higgs doublets that couple to the up and down quark sectors, respectively.

The gauge couplings do not exactly unify in either SM or DHM cases without further assumptions. For illustration, we assumed the unification scale at  $\Lambda = 10^{14}$  GeV. We then calculated the Yukawa coupling matrices at GUT scale  $\Lambda$  by making use of the RGE [17] and taking as initial conditions Eqs. (30a) and (30b) through the relations of Eq. (32) for the SM and Eq. (33) for the DHM.

Within the context of the MSSM, the gauge couplings measured at  $M_Z$  are consistent with a single unified coupling constant at the scale  $\Lambda \simeq 2 \times 10^{16}$  GeV. Assuming  $M_S$  the

natural scale above which the MSSM is valid, the running of the quark Yukawa couplings is done in two steps. First, we run the Yukawa couplings  $h_u$  and  $h_d$  from  $M_Z$  to  $M_S$  by using the RGE for the SM. Then, at the threshold  $M_S$  we match the Yukawa coupling matrices  $Y_u$  and  $Y_d$  of the MSSM with the Yukawa coupling matrices  $h_u$  and  $h_d$  of the SM as

$$Y_u(M_S) = \frac{h_u(M_S)}{\sin \beta}, \quad Y_d(M_S) = \frac{h_d(M_S)}{\cos \beta}. \quad (34)$$

In the numerical analysis the threshold scale  $M_S$  was chosen to be between 1 and 10 TeV. Finally, we run the Yukawa coupling matrices  $Y_u$  and  $Y_d$  from  $M_S$  to  $\Lambda = 2 \times 10^{16}$  GeV and derived the following approximate hierarchical relations for up and down quark Yukawa couplings at GUT scale,

$$y_t : y_c : y_u \approx 1 : \lambda^4 : \lambda^8, \quad (35a)$$

$$y_b : y_s : y_d \approx 1 : \lambda^{2-3} : \lambda^{4-5}, \quad (35b)$$

which are written in terms of powers of the Cabibbo angle,  $\lambda$ , fixed as  $\lambda = 0.22$ . The range of the powers in Eqs. (35b) also reflects the fact we have scanned  $\tan \beta$  from 10 to 50. We would like to remark that these relations also hold for the non-supersymmetric cases (SM and DHM).

The reconstructed Yukawa coupling matrices  $h_u$  and  $h_d$  are presented in Table I for the SM and in Table II for the DHM at the unification scale  $\Lambda = 10^{14}$  GeV by taking into account the scanning of all electroweak input parameters. Rather than to list the numerical values for the Yukawa coupling matrix elements, we have written them in terms of powers of  $\lambda$  [3, 4, 14], considering different WB with zeroes at positions (1, 3), (2, 2) and (2, 3) in the down-quark sector. Since the Yukawa coupling matrices  $h_u$  and  $h_d$  for the DHM depend on  $\tan \beta$ , we have particularized in Table II the cases for  $\tan \beta = 10$  and  $\tan \beta = 50$ .

For the case of MSSM, we show in Table III the Yukawa coupling matrices  $Y_u$  and  $Y_d$  reconstructed at  $\Lambda = 2 \times 10^{16}$  GeV with  $M_S = 1$  TeV for similar WB zeroes. Again, we write the elements of the Yukawa coupling matrices in powers of Cabibbo angle  $\lambda$  considering two values of  $\tan \beta$ : 10 and 50. From the numerical calculations, we have verified that the powers of  $\lambda$  shown in Table III are not much affected by varying the threshold scale  $M_S$  from 1 TeV to 10 TeV. They are however affected by the choice of  $\tan \beta$ , since the down-quark Yukawa matrix increases proportionally with  $\tan \beta$ . In what concerns the up-quark sector the variation is rather smooth and difficult to distinguish. This property remains also valid

TABLE II: The Yukawa matrices  $h_u$  and  $h_d$  in the context of DHM, reconstructed for three different positions of the extra WB zero in  $h_d$ , namely in the positions (1,3), (2,2) and (2,3) at  $\Lambda = 10^{14}$  GeV, and for  $\tan \beta = 10$  and  $\tan \beta = 50$ .

$\tan \beta$	$h_{d13} = 0$	$h_{d22} = 0$	$h_{d23} = 0$
$ h_u $	$\begin{pmatrix} 0 & \lambda^{2.4 - 6.1} & \lambda^{3.3 - 6.6} \\ \lambda^{2.4 - 6.1} & \lambda^{0.5 - 4.9} & \lambda^{0.9 - 7.1} \\ \lambda^{3.3 - 6.6} & \lambda^{0.9 - 7.1} & \lambda^{0.5 - 13.1} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.6 - 8.4} & \lambda^{2.4 - 4.9} \\ \lambda^{3.6 - 8.4} & \lambda^{2.6 - 5.1} & \lambda^{1.6 - 2.7} \\ \lambda^{2.4 - 4.9} & \lambda^{1.6 - 2.7} & \lambda^{0.5} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{4.3 - 9.5} & \lambda^{2.4 - 4.8} \\ \lambda^{4.3 - 9.5} & \lambda^{3.9 - 6.4} & \lambda^{2.6 - 3.3} \\ \lambda^{2.4 - 4.8} & \lambda^{2.6 - 3.3} & \lambda^{0.5} \end{pmatrix}$
	10		
$ h_d $	$\begin{pmatrix} 0 & \lambda^{3.4 - 5.6} & 0 \\ \lambda^{3.4 - 5.6} & \lambda^{1.8 - 7.8} & \lambda^{2.2 - 4.0} \\ 0 & \lambda^{2.2 - 4.0} & \lambda^{1.8 - 6.4} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{4.4 - 6.6} & \lambda^{3.3 - 8.5} \\ \lambda^{4.4 - 6.6} & 0 & \lambda^{3.0 - 3.8} \\ \lambda^{3.3 - 8.5} & \lambda^{3.0 - 3.8} & \lambda^{1.8 - 1.9} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{4.8 - 6.3} & \lambda^{3.4 - 7.5} \\ \lambda^{4.8 - 6.3} & \lambda^{4.3 - 5.7} & 0 \\ \lambda^{3.4 - 7.5} & 0 & \lambda^{1.8 - 1.9} \end{pmatrix}$
$ h_u $	$\begin{pmatrix} 0 & \lambda^{2.3 - 6.2} & \lambda^{3.3 - 6.7} \\ \lambda^{2.3 - 6.2} & \lambda^{0.5 - 6.8} & \lambda^{0.9 - 7.6} \\ \lambda^{3.3 - 6.7} & \lambda^{0.9 - 7.6} & \lambda^{0.5 - 14.9} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.7 - 7.7} & \lambda^{2.4 - 5.0} \\ \lambda^{3.7 - 7.7} & \lambda^{2.7 - 5.5} & \lambda^{1.6 - 2.8} \\ \lambda^{2.4 - 5.0} & \lambda^{1.6 - 2.8} & \lambda^{0.5} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{4.3 - 8.5} & \lambda^{2.3 - 5.0} \\ \lambda^{4.3 - 8.5} & \lambda^{3.9 - 6.8} & \lambda^{2.6 - 3.4} \\ \lambda^{2.3 - 5.0} & \lambda^{2.6 - 3.4} & \lambda^{0.5} \end{pmatrix}$
	50		
$ h_d $	$\begin{pmatrix} 0 & \lambda^{2.1 - 4.4} & 0 \\ \lambda^{2.1 - 4.4} & \lambda^{0.5 - 7.5} & \lambda^{1.0 - 3.4} \\ 0 & \lambda^{1.0 - 3.4} & \lambda^{0.5 - 5.2} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.2 - 5.4} & \lambda^{2.1 - 7.3} \\ \lambda^{3.2 - 5.4} & 0 & \lambda^{1.8 - 2.6} \\ \lambda^{2.1 - 7.3} & \lambda^{1.8 - 2.6} & \lambda^{0.5 - 0.6} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.6 - 5.6} & \lambda^{2.1 - 7.4} \\ \lambda^{3.6 - 5.6} & \lambda^{3.1 - 4.7} & 0 \\ \lambda^{2.1 - 7.4} & 0 & \lambda^{0.5 - 0.6} \end{pmatrix}$

for the case of DHM. We see from our results that the Yukawa power structures vary along the various low effective models and the positions of the WB zeroes. For instance, the range of the matrix element (3,3) for both sectors is really narrow for the cases when the extra WB zero is at the position (2,2) or (2,3) in the down sector. This feature does not depend much on  $\tan \beta$  (DHM and MSSM).

For the case where the WB zero in the down sector is at the position (1,3), the interval of the up-quark Yukawa matrix element (3,3) is large and the matrix element could even be negligibly small. Hence, new possible zeroes with physical implications could be searched. We have found that a new meaningful zero at (3,3) in the up sector could be acceptable for all considered models and independently of  $\tan \beta$ . On the other hand, such possibility does not seem viable in the context of a flavor symmetry, since it would imply that the CKM matrix is derived from large cancellations of up- and down-quark left rotation matrices,  $h_{u33} \ll h_{uij}$ , for  $(i,j) \neq (3,3)$ . A deeper analysis would be needed in order to relate the new texture zeroes emerging from our results with the viable four texture zeroes found in

TABLE III: The Yukawa matrices  $Y_u$  and  $Y_d$  in the context of the MSSM, reconstructed for three different positions of the extra WB zero in  $Y_d$ , namely in the positions (1,3), (2,2) and (2,3) at  $\Lambda = 2 \times 10^{16}$  GeV, with  $M_S = 1$  TeV, and for  $\tan \beta = 10$  and  $\tan \beta = 50$ .

$\tan \beta$	$Y_{d13} = 0$	$Y_{d22} = 0$	$Y_{d23} = 0$	
10	$ Y_u $	$\begin{pmatrix} 0 & \lambda^{2.4 - 6.0} & \lambda^{3.2 - 6.4} \\ \lambda^{2.4 - 6.0} & \lambda^{0.4 - 4.5} & \lambda^{0.8 - 5.9} \\ \lambda^{3.2 - 6.4} & \lambda^{0.8 - 5.9} & \lambda^{0.4 - 12.4} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.7 - 9.1} & \lambda^{2.4 - 4.8} \\ \lambda^{3.7 - 9.1} & \lambda^{2.6 - 4.7} & \lambda^{1.5 - 2.4} \\ \lambda^{2.4 - 4.8} & \lambda^{1.5 - 2.4} & \lambda^{0.4 - 0.5} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{4.3 - 8.5} & \lambda^{2.4 - 4.9} \\ \lambda^{4.3 - 8.5} & \lambda^{3.9 - 6.0} & \lambda^{2.5 - 3.0} \\ \lambda^{2.4 - 4.9} & \lambda^{2.5 - 3.0} & \lambda^{0.4} \end{pmatrix}$
	$ Y_d $	$\begin{pmatrix} 0 & \lambda^{3.5 - 5.7} & 0 \\ \lambda^{3.5 - 5.7} & \lambda^{1.9 - 7.4} & \lambda^{2.3 - 4.1} \\ 0 & \lambda^{2.3 - 4.1} & \lambda^{1.9 - 6.3} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{4.6 - 6.7} & \lambda^{3.5 - 7.7} \\ \lambda^{4.6 - 6.7} & 0 & \lambda^{3.1 - 3.6} \\ \lambda^{3.5 - 7.7} & \lambda^{3.1 - 3.6} & \lambda^{1.9} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{5.0 - 6.4} & \lambda^{3.5 - 7.2} \\ \lambda^{5.0 - 6.4} & \lambda^{4.4 - 5.4} & 0 \\ \lambda^{3.5 - 7.2} & 0 & \lambda^{1.9} \end{pmatrix}$
	$ Y_u $	$\begin{pmatrix} 0 & \lambda^{2.3 - 5.8} & \lambda^{3.2 - 6.5} \\ \lambda^{2.3 - 5.8} & \lambda^{0.3 - 3.7} & \lambda^{0.8 - 7.2} \\ \lambda^{3.2 - 6.5} & \lambda^{0.8 - 7.2} & \lambda^{0.3 - 14.5} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.7 - 8.3} & \lambda^{2.3 - 4.7} \\ \lambda^{3.7 - 8.3} & \lambda^{2.6 - 5.1} & \lambda^{1.5 - 2.6} \\ \lambda^{2.3 - 4.7} & \lambda^{1.5 - 2.6} & \lambda^{0.3 - 0.4} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{4.3 - 8.2} & \lambda^{2.3 - 4.8} \\ \lambda^{4.3 - 8.2} & \lambda^{3.9 - 6.4} & \lambda^{2.5 - 3.2} \\ \lambda^{2.3 - 4.8} & \lambda^{2.5 - 3.2} & \lambda^{0.3 - 0.4} \end{pmatrix}$
	$ Y_d $	$\begin{pmatrix} 0 & \lambda^{2.1 - 4.5} & 0 \\ \lambda^{2.1 - 4.5} & \lambda^{0.5 - 5.4} & \lambda^{0.9 - 2.8} \\ 0 & \lambda^{0.9 - 2.8} & \lambda^{0.5 - 5.2} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.3 - 5.5} & \lambda^{2.1 - 6.3} \\ \lambda^{3.3 - 5.5} & 0 & \lambda^{1.8 - 2.5} \\ \lambda^{2.1 - 6.3} & \lambda^{1.8 - 2.5} & \lambda^{0.5 - 0.6} \end{pmatrix}$	$\begin{pmatrix} 0 & \lambda^{3.7 - 5.2} & \lambda^{2.1 - 6.2} \\ \lambda^{3.7 - 5.2} & \lambda^{3.1 - 4.6} & 0 \\ \lambda^{2.1 - 6.2} & 0 & \lambda^{0.5 - 0.6} \end{pmatrix}$
50				

the literature [3, 14, 15, 18].

The underlying motivation to write the quark Yukawa coupling matrix elements in terms of powers of  $\lambda$  is that such power structure may lead to an insight of the flavor content beyond the Standard Model. At this point, it is clear the relevance of choosing the adequate weak basis where a new symmetry could appear naturally, thus explaining the quark mass and their mixing hierarchies. On the other hand, if a flavor symmetry is responsible for a power structure in the quark Yukawa coupling matrices, it is natural to expect that the smallness of CKM mixing angles should not be due to a relative fine-tuning of up- and down-quark left rotations. Having this criterion of small mixing angles in mind, new patterns of power structures appear at GUT scale. For the case of the SM we obtained approximately

the following structure:

$$|h_u| \propto \begin{pmatrix} 0 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^{\frac{5}{2}} & \lambda \\ \lambda^4 & \lambda & 1 \end{pmatrix}, \quad |h_d| \propto \begin{pmatrix} 0 & \lambda^{\frac{7}{2}} & 0 \\ \lambda^{\frac{7}{2}} & \lambda^{\frac{5}{2}} & \lambda \\ 0 & \lambda & 1 \end{pmatrix}. \quad (36)$$

These results change in the case of the MSSM, we obtained for  $\tan \beta = 10$ ,

$$|Y_u| \propto \begin{pmatrix} 0 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^3 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad |Y_d| \propto \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad (37)$$

and for  $\tan \beta = 50$ ,

$$|Y_u| \propto \begin{pmatrix} 0 & \lambda^5 & \lambda^4 \\ \lambda^5 & \lambda^3 & \lambda \\ \lambda^4 & \lambda & 1 \end{pmatrix}, \quad |Y_d| \propto \begin{pmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & \lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}. \quad (38)$$

These type of power structures could be naturally realized in the context of the Froggatt-Nielsen mechanism [2], where the quark Yukawa couplings arise from non-renormalizable interactions after a scalar singlet field acquires a vacuum expectation value.

#### IV. CONCLUSIONS

With the goal to search for new texture zero structures compatible with the experimental data, we have reconstructed the quark mass matrices at the  $M_Z$  scale in the basis where the matrices are Hermitian and have a maximum of three vanishing elements (one common zero at the same position in the diagonal and an extra zero in either sector). We found that it is unlikely to have more zeroes than the WB zeroes at  $M_Z$  scale. Thus, having a new zero beyond the three WB zeroes implies physical constraints on the parameters. For instance, a “parallel” structure, with zeroes located at (1,1) and (1,3) elements for both quark mass matrices, is not compatible with the electroweak data, which requires  $0.0137 \text{ GeV} \lesssim |M_{u13}| \lesssim 2.58 \text{ GeV}$ .

In addition, we have reconstructed the quark Yukawa couplings in several weak bases with texture zeroes at GUT scale. This was done by considering three low energy models below the GUT scale: SM, DHM and MSSM. Having in mind a gauge flavor symmetry

(Froggatt-Nielsen mechanism) that would explain the hierarchy of quark masses and their mixings, in Tables I, II and III we presented viable power structures of the reconstructed Yukawa coupling matrices as a function of the Cabibbo angle. We showed that if one requires that the smallness of the CKM mixing angles is obtained through small up- and down-quark left rotations, a new pattern of texture zeroes appears. We have also emphasized the importance of right choice of a weak basis where the implementation of the certain flavor symmetry naturally reveals.

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### APPENDIX A: MASSES AND CKM RECONSTRUCTION AT $M_Z$ SCALE

In this appendix, we summarize all the input values needed at  $M_Z$  scale. First, we address the question how to run the quark masses to  $M_Z$  by the QCD RGE programme. Then we present two procedures for the reconstruction of the CKM matrix, starting from three moduli,  $|V_{us}|$ ,  $|V_{ub}|$ ,  $|V_{cb}|$  and either  $\sin 2\beta$  or  $\sin \gamma$ .

To compute the quark running masses at  $M_Z$  we have used the set of RGEs for QCD [13] using a mass-independent subtraction scheme, the  $\overline{MS}$  scheme. The u-, d-, s-quark masses have been estimated at a scale  $\mu \approx 2$  GeV as follows [7]

$$m_u = 2.4 \pm 0.9 \text{ MeV}, \quad (\text{A1})$$

$$m_d = 4.75 \pm 1.25 \text{ MeV}, \quad (\text{A2})$$

$$m_s = 104 \pm 26 \text{ MeV}, \quad (\text{A3})$$

and verifying the relations

$$m_u/m_d = 0.35 - 0.6, \quad (\text{A4})$$

$$m_s/m_d = 17 - 22, \quad (\text{A5})$$

$$(m_u + m_d)/2 = 2.5 - 5.0 \text{ MeV}, \quad (\text{A6})$$

$$(m_s - (m_u + m_d)/2)/(m_d - m_u) = 30 - 50. \quad (\text{A7})$$

For the heavy quark running masses we have used [7]

$$m_c(m_c) = 1.27_{-0.11}^{+0.07} \text{ GeV}, \quad (\text{A8})$$

$$m_b(m_b) = 4.20_{-0.11}^{+0.17} \text{ GeV}, \quad (\text{A9})$$

$$M_t^{\text{pole}} = 171.2 \pm 2.1 \text{ GeV}. \quad (\text{A10})$$

To reconstruct the CKM matrix, some parametrizations seem to be more adequate than others, even though they have no special meaning by themselves. The best evidence for a complex CKM mixing matrix arises from the angle  $\gamma$  [10], and consequently it is a good indicator of the presence of new physics. On the other hand,  $\gamma$  is the least known angle of the unitarity triangle and it is still limited by the statistical and theoretical uncertainties. On the contrary,  $\sin 2\beta$  is measured with precision and it is also more sensitive than  $\gamma$ . Thence, we have decided to reconstruct the CKM mass matrix using  $|V_{us}|$ ,  $|V_{ub}|$  and  $|V_{cb}|$ , while for the CP parameter we have chosen either  $\sin 2\beta$  or  $\sin \gamma$ . Note that these two equivalent sets fully parametrize the CKM unitary matrix. In what follows, we assume the elements  $V_{ud}$ ,  $V_{us}$ ,  $V_{cb}$  and  $V_{tb}$  positive, without loss of generality.

To fully reconstruct the CKM matrix from the input quadruplet  $(V_{us}, V_{ub}, V_{cb}, \sin 2\beta)$  we make use of the Branco-Lavoura (BL) parametrization [19], which depends on  $\lambda$ ,  $A$ ,  $\mu$  and the CP phase  $\phi$ . On the other hand when we reconstruct the CKM matrix from  $(V_{us}, V_{ub}, V_{cb}, \sin \gamma)$  we use the standard parametrization (SP) [20], which is defined by three mixing angles  $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$  and the Kobayashi-Maskawa phase  $\delta$ . As we have mentioned in Section III for both reconstructions we ensure that the value of the CP violating parameter  $J$  is within the experimental range given in Eq. (29).

## 1. Reconstructing CKM with $(V_{us}, V_{ub}, V_{cb}, \sin 2\beta)$

In order to reconstruct the CKM mixing matrix,  $V$ , as a function of  $V_{us}$ ,  $V_{ub}$ ,  $V_{cb}$  and  $\sin 2\beta$  it is useful to use the BL parametrization, which is a Wolfenstein-type parametrization, where the parameters  $\lambda$ ,  $A$  and  $\mu$  are given by

$$\lambda = V_{us}, \quad A = V_{cb}/V_{us}^2, \quad \mu = \frac{|V_{ub}|}{V_{us}V_{cb}}, \quad (\text{A11})$$

and the CP phase is given by  $\phi = -\arg(V_{ub})$ . Once the quantities  $V_{us}$ ,  $V_{ub}$ ,  $V_{cb}$  are given, the parameters  $\lambda$ ,  $A$ ,  $\mu$  are determined. The CP phase  $\phi$  is related to the  $\sin 2\beta$  as,

$$\sin \phi = -\frac{U_{cd}U_{tb}U_{cb}U_{td}}{2V_{us}V_{cs}V_{cb}|V_{ub}|Q} \sin 2\beta, \quad (\text{A12})$$

where  $U_{ij} \equiv |V_{ij}|^2$ . The elements  $U_{cd}$ ,  $U_{td}$  and  $U_{tb}$  are determined by invoking the unitary conditions of  $V$

$$U_{cd} = 1 - U_{cs} - U_{cb}, \quad (\text{A13})$$

$$U_{tb} = 1 - U_{cb} - U_{ub}, \quad (\text{A14})$$

$$U_{td} = U_{cs} + U_{cb} + U_{us} + U_{ub} - 1. \quad (\text{A15})$$

The quantity  $Q$  in Eq. (A12) is then given by

$$Q = \frac{1}{2} (1 - U_{cd} - U_{tb} - U_{cb} - U_{td} + U_{cd}U_{tb} + U_{cb}U_{td}). \quad (\text{A16})$$

Finally,  $V_{cs}$  is written as

$$\begin{aligned} V_{cs} = & [ - (U_{us}U_{cb}U_{ub})^{1/2} \cos \phi + (1 - U_{us} - U_{cb} \\ & + U_{us}U_{cb} - 2U_{ub} + U_{us}U_{ub} + U_{cb}U_{ub} \\ & + U_{ub}^2 - U_{us}U_{cb}U_{ub} \sin^2 \phi)^{1/2} ] / (1 - U_{ub}). \end{aligned} \quad (\text{A17})$$

Therefore, by solving Eq. (A12) we determine  $\sin \phi$ .

## 2. Reconstructing CKM with $(V_{us}, V_{ub}, V_{cb}, \sin \gamma)$

To reconstruct the CKM mixing matrix using  $V_{us}$ ,  $V_{ub}$ ,  $V_{cb}$  and  $\sin \gamma$  we take advantage of the SP, where the CKM matrix is given as a function of the angles  $\theta_{12}$ ,  $\theta_{13}$  and  $\theta_{23}$  through

the relations

$$\sin \theta_{12} = \frac{V_{us}}{\sqrt{1 - |V_{ub}|^2}}, \quad (\text{A18})$$

$$\sin \theta_{13} = |V_{ub}|, \quad (\text{A19})$$

$$\sin \theta_{23} = \frac{V_{cb}}{\sqrt{1 - |V_{ub}|^2}}. \quad (\text{A20})$$

The CP violating phase  $\delta$  can be determined from the input parameters  $V_{us}$ ,  $V_{cb}$ ,  $|V_{ub}|$  and the angle  $\gamma$  by solving the equation

$$\sin^2 \delta = \left[ 1 + \frac{2|V_{ub}|V_{cb}V_{ud}}{V_{us}V_{tb}} \cos \delta + \left( \frac{|V_{ub}|V_{cb}V_{ud}}{V_{us}V_{tb}} \right)^2 \right] \sin^2 \gamma, \quad (\text{A21})$$

where  $V_{ud} = \sqrt{1 - V_{us}^2 - |V_{ub}|^2}$ . We remark that the phases  $\phi$  and  $\delta$  from the BL and SP parametrization can be related using the fact that the CKM matrix element  $V_{cs}$  is real in the BL parametrization and complex in SP,

$$\tan \phi = \frac{\tan \delta}{1 - \frac{V_{cb}V_{us}|V_{ub}|}{V_{tb}V_{ud} \cos \delta}}. \quad (\text{A22})$$

Taking into account the hierarchy of  $V$  we have verified that  $\tan \phi \simeq \tan \delta$  is a good approximation.

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